# Improvement of The Rellich Inequality And Its Applications

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## ABSTRACT

In this paper, we shall study the Rellich inequality

$$\int_{\Omega} |\Delta u|^p dx \geq \left(\frac{n-2p}{p}\right)^p \left(\frac{np-n}{p}\right)^p \int_{\Omega} \frac{|u(x)|^p}{|x|^{2p}} dx \qquad (1)$$

for any $u \in W_0^{2,p}(\Omega)$, where $\Omega$ a bounded domain in $\mathbb{R}^n$ with $0 \in \Omega$, $n \geq 3$, and $1 < p < \frac{n}{2}$. It is known that there is no function $u \in W_0^{2,p}(\Omega)$ for which the best constant $\left(\frac{n-2p}{p}\right)^p \left(\frac{np-n}{p}\right)^p$ is achieved. Hence it is natural to consider that there exist "missing terms" in the right hand side of (1). This talk will present the improvement of the Rellich inequality by adding terms in the right hand side of (1) involving singular weight of type $\left(\log\frac{1}{|x|}\right)^{-2}$. We will show that this weight function is optimal in the sense that the inequality fails for any weight more singular than this one. As an application, the improvement of (1) can be used to analyzed problems related to *p*biharmonic operators.

**KEYWORDS:** Rellich Inequality; biharmonic operators; mathematics; eigenvalue